June 18, 1965

Dr. Juraj Bosak
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Dear Dr. Bosak:

Thank you for your very interesting letter of June 7. I had sent some further remarks to Professor Kotzig which I now fear may have been lost in the mail. However, I am sending exactly the same information in the present letter.

After thinking aobut the problem a little longer, I was able to provide an answer to my own question, namely, a polyhedral graph from which an edge is excluded from availability for any Hamilton circuit. I enclose an example with 24 vertices. From this construction (by further consideration of the operation of replacing a node with the complementary cut graph from a polyhedron which has obligatory edges) one can obtain a polyhedron with no Hamilton circuits which has 38 vertices; this is surely analogous to one that you mention which has also been discovered recently by a student of Professor Victor Klee. This approach has suggested a classification from which we can develop a combinatoric for the construction of polyhedra, based on the level of connectedness of the partial graphs into which the polyhedron can be dissected. I am reasonably certain that the 38-node graph is the smallest non-Hamiltonian polyhedron which has two 3-connected regions. We are now studying graphs with one or zero 3-connected regions, but so far have rather little optimism about finding a smaller non-Hamiltonian graph. My notes on this are waiting to be types and I will send you a copy of a memorandum as soon as this is ready.

May I draw your attention to the enclosed conjecture concerning the symmetry of graphs, to which I have not been able to formulate any approach whatever.

I can understand your difficulty in interpreting my remark about Grace's work: that he may have rejected some construction on the ground that it was isomorphic to a construction already within his list. However, his condition of equi-surroundedness is not a rigorously sufficient basis for isomorphism. Therefore, his list may be incomplete in respect of some graph which is equi-surrounded but is not isomorphic with an item that is in the list.

You ask about the connection between these problems and my research. The only connection is my hope to be able to anticipate all possible cyclic nets which may then have realizations as organic molecular structures. The enumeration of Hamilton circuits provides by far the most convenient approach to generating and encoding the set of structures, as well as for describing them in some canonical form. The properties of the graphs with respect to Hamilton circuits then became quite fundamental. I have probably overemphasized the role of the polyhedra among

the totality of cyclic nets. However, it was just the non-Hamiltonian polyhedra that posed the most enigmatic questions with respect to the possibility of anticipating the entire series. I would certainly admit that the abstract analyses go rather beyond the most immediate requirements of any chemical enumeration of practical extent. However, I would also have to say that these manipulations have taught me a great deal about some elementary aspects of graph theory and that I am very grateful for the insights.

I should allude again to Professor Klee's interest in the same subject, and suggest that you might wish to be in more direct correspondence with him.

Yours cordially,

Joshua Lederberg
Professor of Genetics